Exercises on Market structures Answers

Exercise 1: Monopoly

Question 1: The monopolist profit is:

$$\Pi = (32 - q) \cdot q - 4q$$

Maximizing profit with the derivative yields: $q^* = 14$. Then the optimal price is $p^* = 18$, and the monopolist profit is $\Pi = 196$.

Question 2: The monopolist profit is now:

$$\Pi = (60 - 2q) \cdot q - 4q$$

Maximizing profit with the derivative yields: $q^* = 14$. Then the optimal price is $p^* = 32$, and the monopolist profit is $\Pi = 392$.

Exercise 2: Monopoly

The inverse demand function is: $p = 20 - \frac{x}{200}$.

Hence total revenue is $R(x) = 20x - \frac{x^2}{200}$ and profit is $\Pi(\mathrm{x}) = 20x - \frac{x^2}{200} - 5x - 10{,}000$.

The derivative is null for $x = 1500$. The price is then $p = 20 - \frac{1500}{200} = 12.5$ (dollars) and the maximum profit is $\Pi(1500) = 15 \cdot 1500 - \frac{1500^2}{200} - 10{,}000 = 1{,}250$ dollars.

Exercise 3: perfect competition

In a perfectly competitive market, the firm is price-taker: $p^* = 50$.

The optimal quantity can be computed using the optimality condition $p = MC$.

$$50 = 2 \cdot q$$
$$q^* = 25$$

Exercise 4: taxation

Question 1: From Q = 20 – P and Q = 3P, equate the two to solve for equilibrium price and quantity at P* = 5 and Q* = 20 – 5 = 15.

Question 2: Rewrite the demand and supply equation as P = 20 – Q and P = Q/3. With $4 tax on producers, the supply curve after tax is P = Q/3 + 4.
Hence, the new equilibrium quantity after tax can be found from equating P = Q/3 + 4 and P = 20 – Q, which gives $Q_T = 12$.
Price producers receive is from pre-tax supply equation $P_{net} = Q_T/3 = 12/3 = 4$.
Price consumers pay is obtained from demand equation $P_T = 20 – Q_T = 20 – 12 = 8$. It can also be obtained from taxed supply equation: $P_T = Q_T/3 + 4 = 12/3 + 4 = 8$.

Question 3: Government revenue is given by tax times the quantity transacted in the market so $4 x 12 = $48.

Question 4: Deadweight loss is calculated from ½ x $4 x (15 – 12) = $6, of which $4.5 is from consumer's under-consumption, and $1.5 is from producer's under-production.

Exercise 5: quantity quota

Question 1: From P = 200 – Q/10 and P = 20 + Q/20, equate the two to solve for equilibrium quantity: 200 – Q/10 = 20 + Q/20, so: Q* = 1,200 and equilibrium price is P* = 20 + 1200/20 = $80.

Question 2: With quantity restricted at 600 statisticians, the willingness to pay for a statistician is P = 200 – 600/10 = $140. This is the wage that statistician receives. Since quota is binding, only 600 statisticians are employed.

Question 3: Note that at quantity Q = 600, the willingness to supply is at wage P = 20 + 600/20 = $50. Deadweight loss is calculated from ½ x (1,200 – 600) x ($140 – $50) = $27,000.

Exercise 6: price floor

Question 1: From P = 100 – Q/100 and P = Q/100, equate the two to solve for equilibrium quantity: 100 – Q/100 = Q/100, so: Q* = 5,000 and equilibrium price is P* = 5000/100 = $50. Price floor – government buys surplus (excess supply) and pays storage cost

Question 2: At price P = 60, consumers are willing to buy only Q = 4,000 solved from 60 = 100 – Q/100. However, producers are supplying Q = 6,000 solved from 60 = Q/100. Hence, the government must by 6,000 – 4,000 = 2,000 units.

Exercise 7: two-part tariff

Monopoly sells its product to N = 10 identical consumers each of whom has individual demand P = 30 − 2q, where q is quantity demanded by a single consumer at price P. The firm has constant marginal cost MC = 5 and no fixed cost.

Question 1: At price P each consumer demands q = 30−P 2 units, therefore, market quantity demanded is $Q = 10q = 10\frac{30-P}{2}$, which solves for inverse market demand P = 30−0.2Q.
π = TR − TC = P · Q + AC · Q.
Maximizing profits (or setting MR = MC), we get that the profits are maximized at QM = 62.5.
Substitute into demand to find PM = 17.5.
Calculate profits π = 781.25, CS=390.625.

To find welfare loss you need the efficient output, which is where the MC curve intersects the demand curve: P = 30 − 0.2Q = 5 = MC, denote Q∗ = 125.
The deadweight loss is then: DWL=390.625

Question 2:
When consumers have identical demands, the optimal two-part pricing is to set per unit price P = MC, which is 5 in this case, and then charge the fixed fee equal to CS at that price.
Use individual demands to find that at P=5, each consumer demands q=12.5. For each consumer the surplus is then CS=156.25, which is the highest fee he is willing to pay to be able to purchase the good at P=5.
Notice that in this case the firm does not make any profits on the unit sales as P=AC, so the firm's profits come from the fixed fees collected, which totals 1,562 for the 10 consumers. The consumer's surplus is nil; as it is entirely captured by the firm.

Exercise 8: third-degree price discrimination

<u>Question 1:</u> This would be a case of ordinary price discrimination: segregate consumers into market segments and charge different prices in each segment. Find inverse demands in both markets $P_A = 50 - 0.5Q_A$ and $P_B = 20 - 0.25Q_B$. Profits are the difference between the sum of the revenues generated in both market segments and the total cost of producing the output:

$$\pi(Q_A, Q_B) = TR_A(Q_A) + TR_B(Q_B) - TC(Q_A + Q_B)$$

$$\pi(Q_A, Q_B) = P_A \cdot Q_A + p_B \cdot Q_B - TC(Q_A + Q_B)$$

$$\pi(Q_A, Q_B) = (50 - 0.5Q_A) \cdot Q_A + (20 - 0.25Q_B)Q_B - \frac{(Q_A + Q_B)\text{^}2}{12}$$

Finding the optimum profit means maximizing profit for Q_A and Q_B. The first-order conditions (FOC) are:

$$\frac{\partial \pi}{\partial Q_A} = 50 - Q_A - \frac{Q_A + Q_B}{6}$$
$$\frac{\partial \pi}{\partial Q_B} = 20 - 0.5Q_B - \frac{Q_A + Q_B}{6}$$

In other words, we have $MR_A = MC = MR_B$. The quantities that satisfy both FOCs are $Q_A = 40$, $Q_B = 20$ and the respective prices are $P_A = 30$ and $P_B = 15$.

<u>Question 2:</u>
By definition, the elasticity is: $\varepsilon = \frac{dq}{dp} \cdot \frac{p}{Q}$

In Segment A, $\frac{dQ_A}{dp_A} = -2$, so $\varepsilon_A = -2 \cdot \frac{30}{40} = -1.5$.
In Segment B, $\frac{dQ_B}{dB} = -4$, o $\varepsilon_B = -4 \cdot \frac{15}{20} = -3$.

Exercise 9: Two-part tariff and second-degree price discrimination

Question 1: For consumers of type 1 the package is $q_l = 5\,; F_l = 5$.
For consumers of type 2 the package is $Q_H = 10\,; F_H = 50$.
Similar to the two-part pricing, the firm will choose p = MC = 0 and charge the fee Fi equal to the total value (area under the demand curve) for the package.

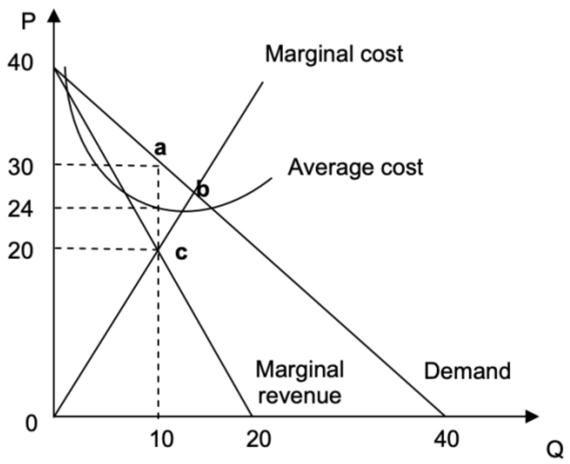
Question 2: Now consumers of type 1 keep buying their package (for them the other package does not look attractive at all), but consumers of type 2 prefer the package designed for type 1: If they consume 5 units they get total value TV = 37.5 (area under their D-curve for q=5), but only pay 12.5 dollars, which leaves them with a positive consumer surplus CS = 25, whereas their own package results in CS = 0.

Question 3: If the firm wants to sell $q_L = 5$ and $q_H = 10$ respectively, it should lower FH at $25 so that the customers of this type get at least as high CS as when they buy the other package.

Exercise 10: Market power

a) MR = -2Q + 40

b) The graph is:

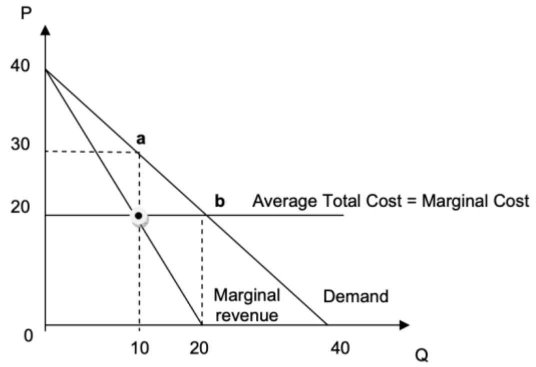


c) Using MR=MC, we have -2Q + 40 = 2Q, and we can get $Q_M = 10$
Plugging $Q_M = 10$ into demand equation, we have $P_M = -10 + 40 = \$30$

d) $CS_M = 10 \text{ x } 10/2 = \$50$
$PS_M = 10 \text{ x } 10 + 20 \text{ x } 10/2 = \$200$

By definition, Profits = TR – TC
$TR = P_M \text{ x } Q_M = 30 \text{ x} 10 = \$300$
$TC = Q^2 + 140 = 102 + 140 = \$240$
Then, Profits = TR – TC = \$300 - \$240 = \$60

e) Using MR = MC, we have -2Q + 40 = 20, and we can get Q_M = 10
Plugging Q_M = 10 into demand equation, we have P_M = -10 + 40 = $30



f) The competitive market equilibrium price should satisfy P = MC, so P_{pc} = $20
Plug P_{pc} = 20 into demand, we get 20 = -Q + 40, so Q_{pc} = 20.

g) CS(monopoly) = (1/2)x($40 per unit - $30 per unit) x (10 units) = $50
CS(perfect competition) = (1/2) x ($40 per unit - $20 per unit) x (20 units) = $200
The consumer surplus is $150 greater with perfect competition than with monopoly.

h) PS(monopoly) = 10x10 = $100
PS(perfect competition) = $0
The producer surplus is $100 greater with monopoly than with perfect competition

i) TS(perfect comp.) = CS(perfect competition) + PS(perfect competition) = 200 + 0 = $200
TS monopoly = CS(monopoly)+PS(monopoly) = 50+100 = $150

DWL = [TS(perfect competition)] - [TS (monopoly)]
DWL = 200 – 250 = $50

Alternative way of calculating: graphically DWL (with monopoly) = (1/2)($30 per unit - $20 per unit)(20 units – 10 units) = $50